

Warm-Up:

Find the sum of the zeros of the function: $f(x) = x^3 - 10x^2 + 11x + 70$.

1. Given the differential equation

$$\frac{dy}{dx} = 128xy \text{ with the initial condition}$$

$$y(0) = 12, \text{ find } y\left(\frac{1}{8}\right) \text{ in terms of}$$

fundamental mathematical constants.

2. Jill happens upon a ladder that is about 15 feet long resting against a wall. All of a sudden it begins to slide! Being the quick genius she is, Jill knows that the bottom is sliding away from the wall at 0.5 ft/sec and started 10 feet from the wall. How fast will the ladder be falling down the wall after 8 seconds?

3. Jack is carrying a pail of water up a hill and remembers his favorite polar function:

$r = 3\sin(3\theta)$. He knows it's a pretty flower in the polar plane that looks like it has petals. What is the area of one of these petals in terms of fundamental mathematical constants?

4. Given the implicit equation $y^2x + x^2 = 2$, find the rate of change of y with respect to x when $x = 1$.

5. Evaluate the integral: $\int_0^{10} \sin^2\left(\frac{\pi x}{5}\right) dx$

6. Evaluate the limit $\lim_{n \rightarrow \infty} \left(1 + \frac{7}{2n}\right)^{3n}$ in terms of fundamental mathematical constants.

7. Rory runs on a line for fun. His x -value at any given time, $t \geq 0$, is shown from the function: $R(t) = x^3 - 9x^2 + 20x - 12$. Rory changes direction sometimes and might stop for a slight moment. What is the sum of the times where Rory stops and changes direction?

8. What is the derivative of x^x in terms of x ?

9. Find the area of the region enclosed by $y = 2xe^{x^2}$ and $y = -3x$ and $x = 1$ in terms of fundamental mathematical constants.

10. A parametric function $r(t)$ describes how a fly named Jack moves on the x-y plane. If $r(t) = \langle t \sin(t), t^2 \rangle$, what is the slope of the tangent line to the path when Jack has been on the move for π seconds?

TB1. What is the area of half of the rose petals in the curve $r = \cos(4\theta)$?

TB2. If the integral is convergent, what is the value of the integral $\int_{-\infty}^{\infty} 10xe^{-x^2} dx$ if not state it is divergent?

Answers:

1. $12e$
2. $-7/\sqrt{29} = -7\sqrt{29}/29$
3. $3\pi/4$
4. $-3/2$
5. 5
6. $e^{(21/2)}$
7. 6
8. $x^x \ln(x) + x^x$
9. $1/2 + e$
10. -2
11. $\pi/4$
12. 0