

1. If $f(x) = \begin{cases} 1 + 4x - x^2, & x \geq 0 \\ mx + b, & x < 0 \end{cases}$ and f is differentiable at $x = 0$, then

- a) $m = 1, b = 2$ b) $m = 2, b = 0$ c) $m = 4, b = 1$ d) $m = 2, b = 1$

2. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} =$

- a) ∞ b) 1 c) $\frac{3}{2}$ d) $\frac{2}{3}$

3. $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln(2)}{h} =$

- a) 1 b) 0 c) $\frac{1}{2}$ d) 2

4. Which of the following is an antiderivative of $x^2 \cos(x^3)$?

- a) $\frac{1}{3} \sin(x^3)$ b) $\frac{1}{3} x^3 \sin(x^3)$ c) $-\frac{1}{3} \sin(x^3)$ d) $-x^2 \sin(x^3) + 2x \cos(x^3)$

5. $\lim_{x \rightarrow \infty} \arctan x =$

- a) 1 b) ∞ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

6. Evaluate the integral below:

$$\int_0^4 \frac{x}{x^2 + 9} dx$$

- a) $\ln\left(\frac{5}{3}\right)$ b) $\ln(5)$ c) $\ln\left(\frac{5}{9}\right)$ d) $\ln(25)$

7. If $\frac{x+1}{x^3+x^2-6x}$ is decomposed into partial fractions, then the coefficient of $\frac{1}{x+3}$ is

- a) $-\frac{2}{15}$ b) $-\frac{1}{6}$ c) 0 d) $\frac{3}{10}$

8. Identify the interval below where the function $f(x) = -5x^3 + 7x^2 + x - 6$ is concave down.

- a) $(-\infty, \infty)$ b) $(-\infty, \frac{7}{15})$ c) $(-\frac{7}{15}, \infty)$ d) $(\frac{7}{15}, \infty)$

9. A rectangle has adjacent sides on the positive x and y axes and a vertex on the line $y = 1 - \frac{x}{2}$. What is the maximum area of such a rectangle?

- a) 1 b) 2 c) 4 d) $\frac{1}{2}$

10. The function $f(x) = ax^2 + \cos(x)$ has an inflection point at the point with x -coordinate π . What is the value of a ?

- a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{\sqrt{2}}{4}$

11. Evaluate the integral below:

$$\int_0^4 \frac{x}{\sqrt{x^2 + 9}} dx$$

- a) $\frac{1}{2} \ln\left(\frac{5}{3}\right)$ b) 4 c) $\frac{1}{2} \ln(5)$ d) 2

12. If $f'(2) = 4$ and if $g(x) = f(x^2 - 2)$, then $g'(2) =$

- a) 4 b) 8 c) 0 d) 16

13. $\lim_{x \rightarrow 2} \frac{2x^2 - 8}{x - 2} =$

- a) 4 b) 8 c) ∞ d) $-\infty$

14. If $y + \tan(xy) = 0$, then $\frac{dy}{dx} =$

a) $-\sec^2(xy)$

b) $-\frac{y \sec^2(xy)}{1+x \sec^2(xy)}$

c) $-\frac{x}{\sec^2(xy)}$

d) $\frac{\sec^2(xy)}{1+x \sec^2(xy)}$

15. If $f(x) = \int_0^x \cos(t^2) dt$, then $f'(x) =$

a) $\sin(x^2)$

b) $\int_0^{x^2} \cos(t^2) dt$

c) $\cos(x^2)$

d) $\int_0^x \sin(t^2) dt$

16. Find the general solution to the differential equation $x dy - y dx = 0$.

a) $y = e^x + C$

b) $y = Cx$

c) $y = x + C$

d) Can't be solved

17. What is the y -intercept of the line which is tangent to the graph of $y = 2x^2 - x + 1$ at the point where $x = 1$?

a) -3

b) 1

c) 0

d) -1

18. Let A be the region bounded by $y = \ln x$, the x -axis, and the line $x = e$. Which of the following represents the volume of the solid generated when A is revolved around the y -axis?

a) $\pi \int_0^1 (e^2 - e^{2y}) dy$

b) $\pi \int_0^1 (e^2 - e^y) dy$

c) $\pi \int_0^1 e^{2y} dy$

d) $2\pi \int_1^e (e^2 - e^{2y}) dy$

19. When the value of $\cos 2$ is approximated by using the 4th degree Taylor polynomial about $x = 0$, the value of $\cos 2$ is

a) $1 + \frac{4}{2} - \frac{16}{24}$

b) $1 + \frac{4}{2} + \frac{16}{24}$

c) $1 - \frac{4}{2} - \frac{16}{24}$

d) $1 - \frac{4}{2} + \frac{16}{24}$

20. Identify the interval(s) below where the function $f(x) = x\sqrt{20 - x^2}$ is increasing.

- a) $(\sqrt{10}, \infty)$ b) $(-\sqrt{10}, \sqrt{10})$ c) $(-\infty, \sqrt{20})$ d) $(-\sqrt{20}, -\sqrt{10}) \cup (\sqrt{10}, \sqrt{20})$

21. The slope of the line normal to the curve $h(x) = 2 \cos 4x$ at $x = \frac{\pi}{12}$ is

- a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $\frac{1}{4}$

22. Given $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$ when $t = 0$ and $s = 1$. Find t when $s = \frac{3}{2}$.

- a) $\frac{1}{2}$ b) $\frac{\pi}{2}$ c) 1 d) $\frac{2}{\pi}$

23. The function $f(x) = \sqrt[3]{4 - x^2}$ has a vertical tangent line at

- a) $x = 0$ b) $x = 2, -2$ c) $x = 1, -1$ d) $x = 0, 2, -2$

24. $\lim_{x \rightarrow 0} \frac{1 - \cos^2(2x)}{x^2} =$

- a) -2 b) 0 c) 2 d) 4

25. Evaluate the integral below:

$$\int_{-1}^2 \frac{|x|}{x} dx$$

- a) -3 b) 1 c) 2 d) 3

26. The derivative of $(1 + x)^n \cdot \ln(1 + x)$ with respect to $(1 + x)$ is

- a) $(1 + x)^{n-1}[1 + n \ln(1 + x)]$ b) $1 + \ln(1 + x)$ c) $n \ln(1 + x)^n$ d) $\frac{1}{n} + \ln(1 + x)$

27. What is the area enclosed by one leaf of the graph with polar equation $r = \sin(3\theta)$?

- a) $\frac{\pi-3}{24}$ b) $\frac{\pi}{12}$ c) $\frac{\pi-3}{12}$ d) $\frac{1}{3}$

28. The region bounded by $y = \sqrt{x^2 - 1}$, $y = 0$, $x = 1$, and $x = 3$ is revolved about the x -axis. What is the volume of the resulting solid?

- a) 8π b) $\frac{8\pi}{3}$ c) $\frac{16\pi}{3}$ d) $\frac{20\pi}{3}$

29. Evaluate the integral below:

$$\int_1^3 4x\sqrt{6-2x} \, dx$$

- a) $-\frac{16}{3}$ b) $\frac{19}{3}$ c) $\frac{48}{5}$ d) $\frac{96}{5}$

30. Find the length of the curve described by $x = \arcsin(t)$ and $y = \frac{1}{2}\ln(1 - t^2)$ for $0 \leq t \leq \frac{1}{2}$.

- a) $2\sqrt{3}$ b) $2 \ln \sqrt{3}$ c) $2 + 3 \ln \sqrt{3}$ d) $\frac{1}{2} \ln 3$